



AI and Game Theory

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Machine Learning Methods

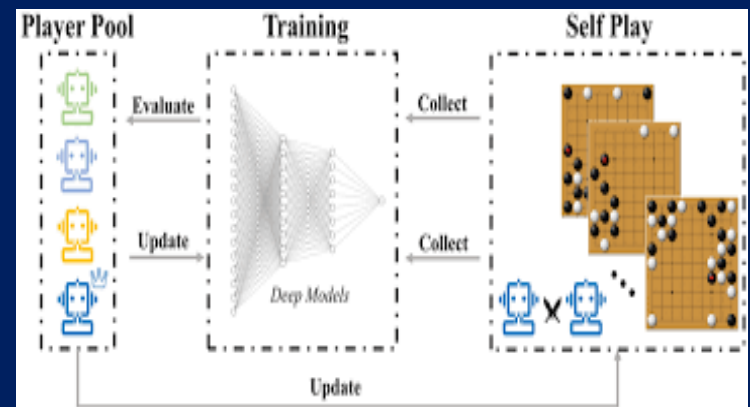
1. Traditional Chess Engines (Deep Blue, Stockfish, Komodo):

- They use hand-coded evaluation functions. These functions focus on material advantage (the value of the pieces), position, and other strategic factors.
- Basically, human programmers define how the engine evaluates each chess position (Brutal Force).



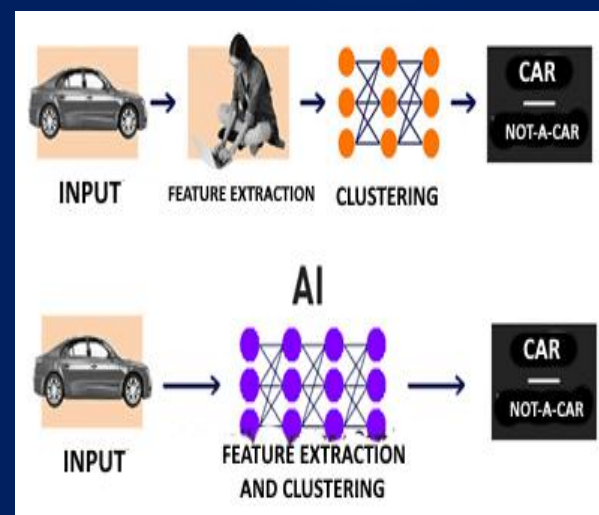
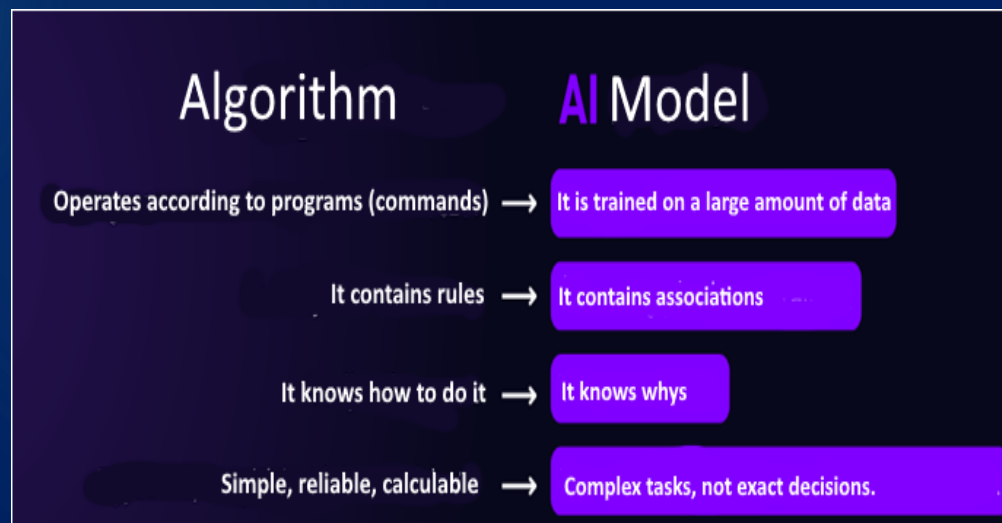
2. Self-learning AI (AlphaZero, Leela Chess Zero):

They use deep neural networks and play millions of games against themselves. Through this, they figure out the best moves on their own, instead of following rules written by human chess masters.



What is ChatGPT?

- ▶ **Large Language Models (LLM):** An AI-based system capable of understanding and generating human language texts through studying large amounts of text.
- ▶ Not an algorithm, but a complex language model.
 - ✓ ChatGPT does not respond to questions based on a simple set of rules, but has learned from billions of texts and contexts, and is able to generate responses that reflect a deeper understanding of the context and meaning behind the input question.



What is ChatGPT?

1. Traditional Language Process:

Input:

"The cat is in the ...,"

Outputs:

- "The cat is in the garden is"
- "The cat is in the house is"

Formal description in classical grammar:

Rules:

- S → "The cat is " + "in/on" + N
- N → "garden"
- N → "house"

2. Self-learning AI:

The deep learning model selects the continuation based on a probability distribution learned from the data:

$$P(\text{in the garden} \mid \text{The cat is}) = 0.65$$

$$P(\text{in the house} \mid \text{The cat is}) = 0.25$$

$$P(\text{in the attic} \mid \text{The cat is}) = 0.07$$

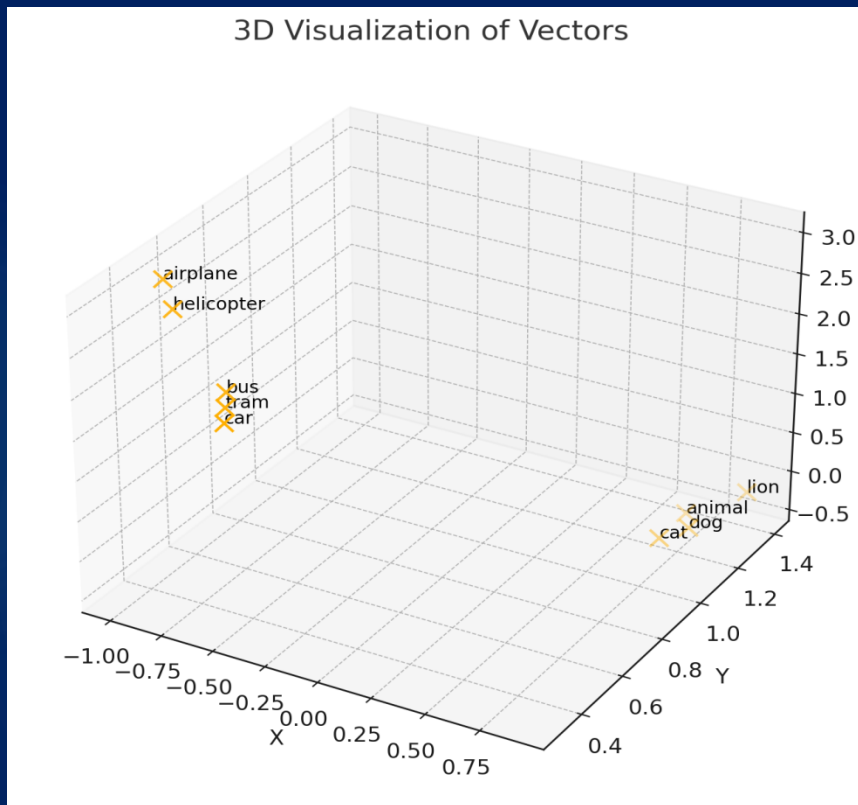
$$P(\text{on the tree} \mid \text{The cat is}) = 0.03$$

Possible outputs (and their generation probabilities):

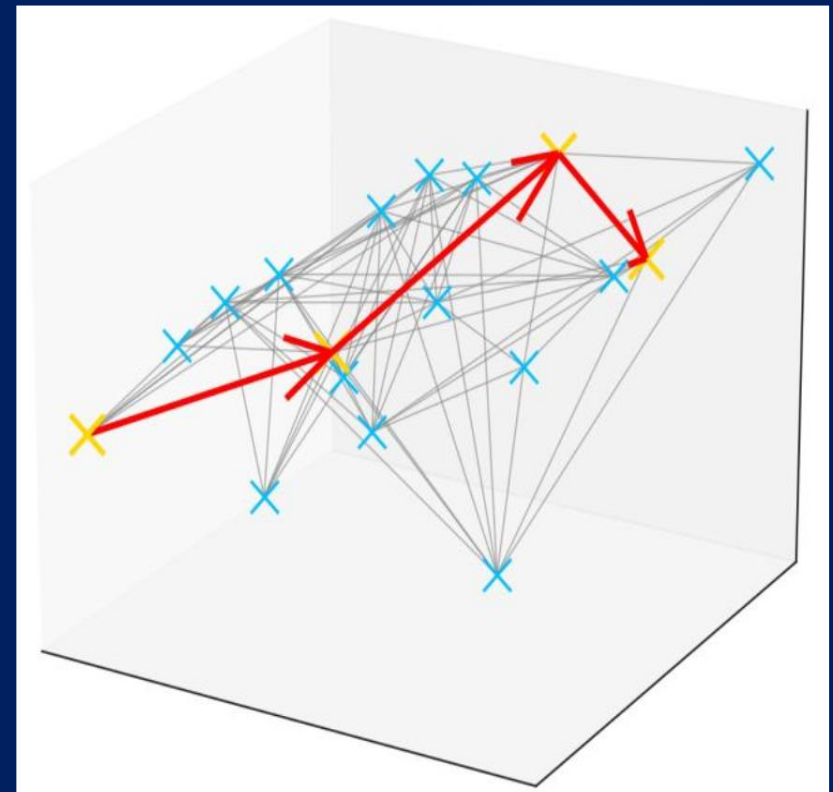
- "The cat is in the garden." (65%)
- "The cat is in the house." (25%)
- "The cat is in the attic." (7%)
- "The cat is on the tree." (3%)

What is ChatGPT?

2. Self-learning AI:



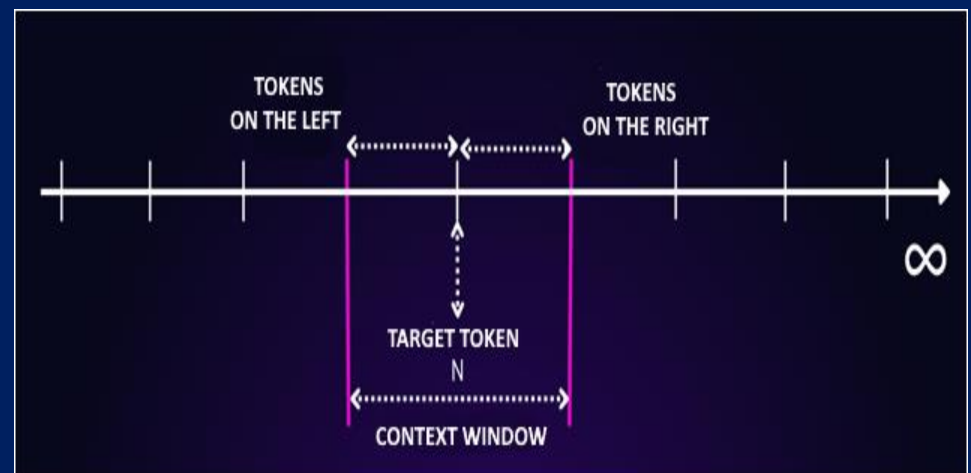
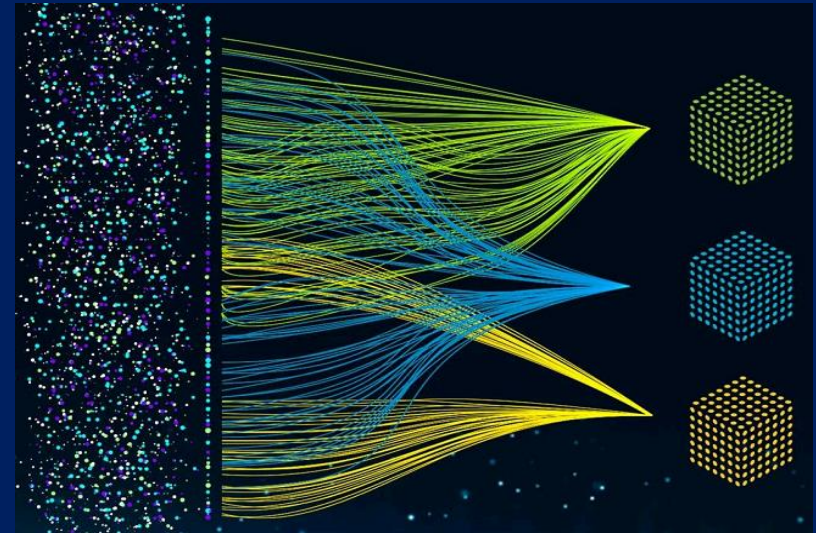
Representations of meanings



The processing of information

Tokenization and Context Window

- ▶ **Basis of Operation: Pattern Recognition.**
- ▶ **Tokenization:** Tokens are smaller units of text (e.g., words).
- ▶ **Context Window:** The maximum number of tokens the model can process at once when generating a response.
 - ▶ It is essential that the most important information stays within the context window, so that the AI can accurately interpret and present it.



Warning!

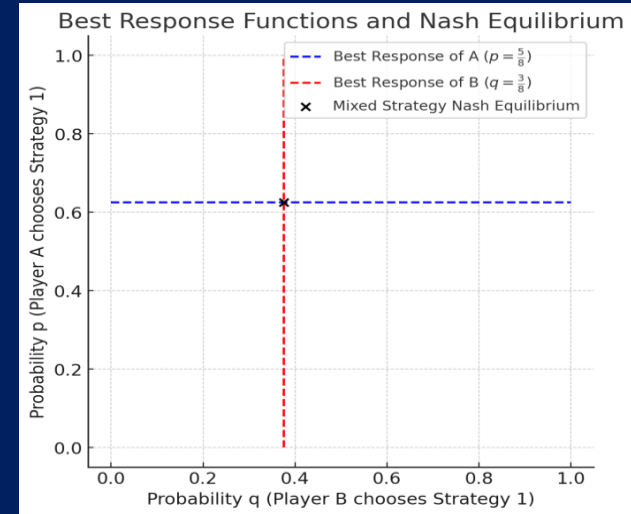
➤ ChatGPT has limitations.

Hallucination when the model generates non-existent or inaccurate information, as if it were real.

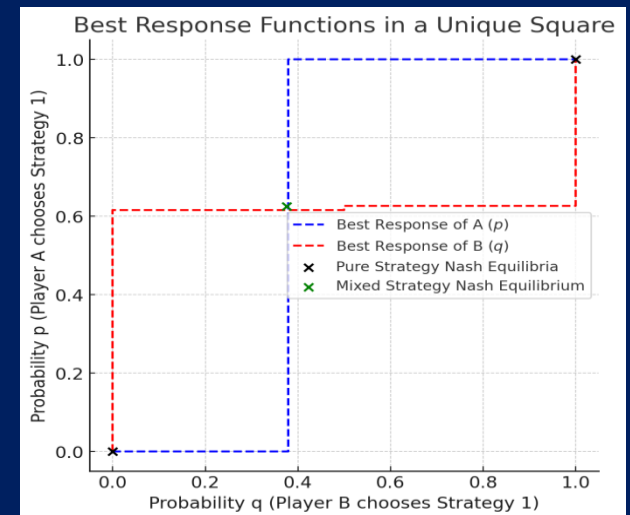
➤ It is important to validate the answers and place the model's results in context.

✓ It should not be used uncritically in "Copy/Paste" mode; one must understand what is happening "under the hood".

Display the result by Best Response Function in a unique square.



Best response function **is a step function** in a unique square



Giving Commands: Minimum Prompt

I have a 2x2 matrix game.

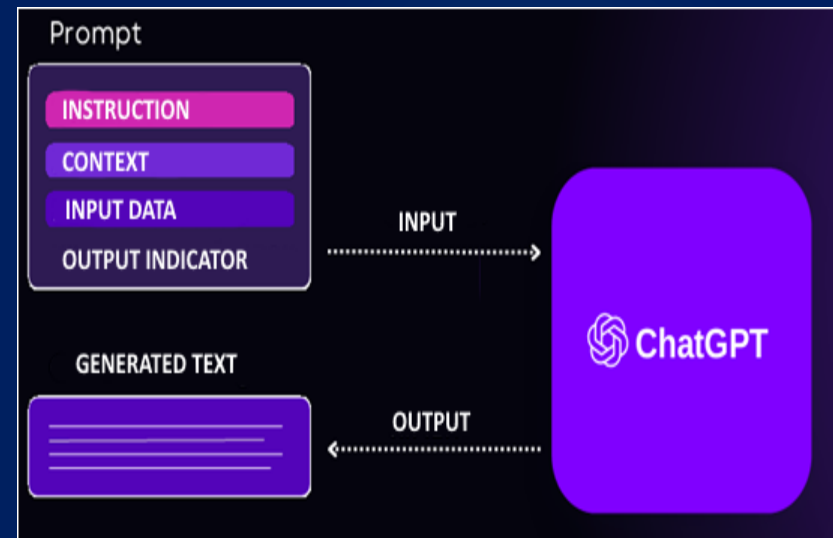
Player A's payoff matrix is: $[[a_{11}, a_{12}], [a_{21}, a_{22}]]$.

Player B's payoff matrix is: $[[b_{11}, b_{12}], [b_{21}, b_{22}]]$.

- *Find all pure strategy Nash equilibria (with step-by-step best response analysis).*
- *Compute the mixed strategy Nash equilibrium, showing all calculations clearly.*
- *Plot the best response functions in a unique square, using step functions.*
- *Mark all intersection points on the plot and display their coordinates.*

Prompt Scheme Design

- ▶ **One of the fundamental elements of using ChatGPT is giving correct and well-formulated commands (prompts):**
- ▶ **Role Assignment:** During role assignment, we specify the "role" in which ChatGPT should operate.
- ▶ **Instruction:** This is a specific task or request that the AI model needs to execute.
- ▶ **Input Data:** Input data consists of the information or questions for which we are seeking answers.
- ▶ **Output Indicator:** Output indicators define the form or structure in which we expect the response (e.g., summary, table, a specific answer).



Prompt Scheme Design

- 1. Role Assignment:** Step into the role of an **experienced game theorist**, specializing in solving 2×2 strategic matrix games. Your task is to systematically analyze the game, identifying all equilibrium points and strategic insights.
- 2. Instruction:** Analyze the provided 2×2 payoff matrices for both players. Follow these steps:
 - ▶ **Identify all Pure Strategy Nash Equilibria (PSNE)** using the best response method.
 - ▶ **Calculate the Mixed Strategy Nash Equilibrium (MSNE)** by making each player indifferent between their strategies.
 - ▶ **Graphically illustrate** the Best Response Functions (BRFs) in a unit square.
 - ▶ **Mark and label all equilibrium points** (both pure and mixed) on the graph.
- 3. Context:** The game represents a strategic interaction between two rational players, each having two available strategies. Both players aim to maximize their payoffs, considering the other player's potential choices. The analysis should reveal:
 - ▶ Whether dominant strategies exist.
 - ▶ Where players reach equilibrium (stable outcomes).
 - ▶ How mixed strategies may be optimal when no pure strategy equilibrium is strictly preferred.

Prompt Scheme

4. Input Data:

- ▶ **Action Trigger:** The user uploads /types 2X2 payoff matrices for both players:
 - ❑ Player A's payoff matrix:
[[a11, a12], [a21, a22]]
 - ❑ Player B's payoff matrix:
[[b11, b12], [b21, b22]]
- ▶ **Use the Data:**
 - ❑ Apply the **best response method** to find pure strategy equilibria.
 - ❑ Solve for mixed strategies by equating expected payoffs.
 - ❑ Plot the BRFs as step functions in a unit square.
 - ❑ Label intersection points with their coordinates.

Prompt Scheme

5. Output Indicator:

▶ Table of Equilibria:

Equilibrium Type	Player A Strategy	Player B Strategy	Coordinates (q, p)
Pure Strategy Nash Equilibrium #1	Strategy X	Strategy Y	(,)
Pure Strategy Nash Equilibrium #2	Strategy Z	Strategy W	(,)
Mixed Strategy Nash Equilibrium	Mixed (probability)	Mixed (probability)	(,)

▶ Graph:

- ❑ Display BRFs as step functions.
- ❑ Mark and label all equilibrium points in the unit square.

▶ Short Report:

- ❑ Summary of key findings.
- ❑ Intuitive explanation of why the equilibrium points are stable.
- ❑ Interpretation of mixed strategies (e.g., when no dominant strategy exists).

Minimum Prompt

I have a 2x2 matrix game.

Player A's payoff matrix is: $[[a_{11}, a_{12}], [a_{21}, a_{22}]]$.

Player B's payoff matrix is: $[[b_{11}, b_{12}], [b_{21}, b_{22}]]$.

- 1. Find all pure strategy Nash equilibria (with step-by-step best response analysis).*
- 2. Compute the mixed strategy Nash equilibrium, showing all calculations clearly.*
- 3. Plot the best response functions in a unique square, using step functions. Mark all intersection points on the plot and display their coordinates.*
- 4. Give a short report for the outcome*

I have a 2x2 matrix game.

Player A's payoff matrix is: $[[4, -1], [-1, 2]]$.

Player B's payoff matrix is: $[[2, -1], [-1, 4]]$.

- Find all pure strategy Nash equilibria (with step-by-step best response analysis).*

1. Pure Strategy Nash Equilibria:

Step 1: Best Responses

If Player B chooses Strategy 1:

Player A payoff:

Strategy 1 \rightarrow 4

Strategy 2 \rightarrow -1

\Rightarrow Best Response: Strategy 1.

If Player B chooses Strategy 2:

Player A payoff:

Strategy 1 \rightarrow -1

Strategy 2 \rightarrow 2

\Rightarrow Best Response: Strategy 2.

If Player A chooses Strategy 1:

Player B payoff:

Strategy 1 \rightarrow 2

Strategy 2 \rightarrow -1

\Rightarrow Best Response: Strategy 1.

If Player A chooses Strategy 2:

Player B payoff:

Strategy 1 \rightarrow -1

Strategy 2 \rightarrow 4

\Rightarrow Best Response: Strategy 2.

I have a 2x2 matrix game.

Player A's payoff matrix is: $[[4, -1], [-1, 2]]$.

Player B's payoff matrix is: $[[2, -1], [-1, 4]]$.

✓ Pure Strategy Nash Equilibria:

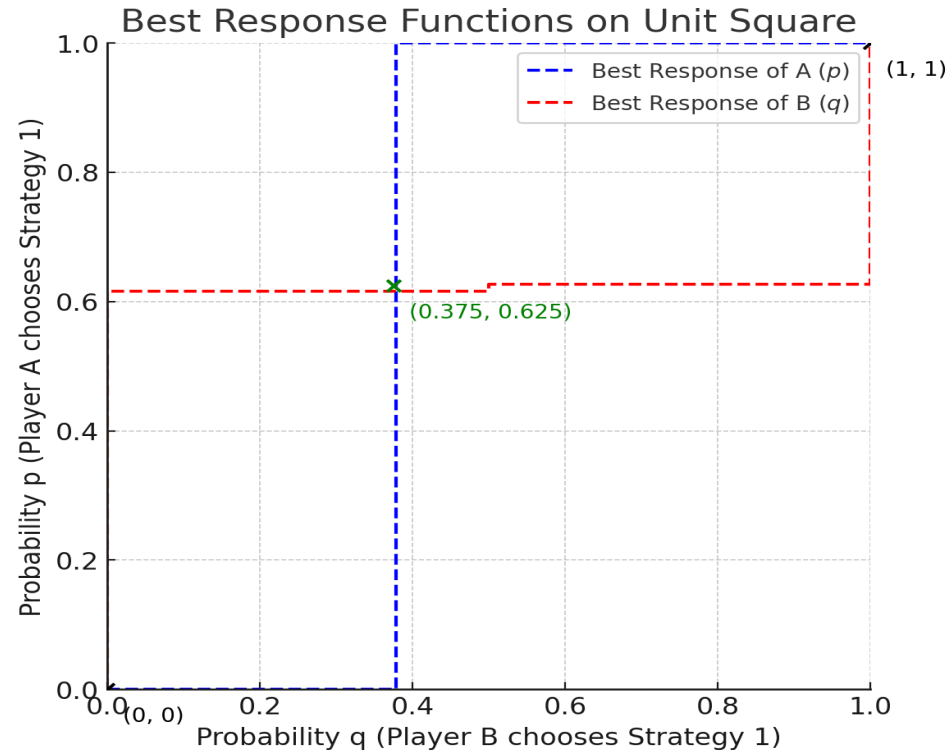
Equilibrium Type	Player A Strategy	Player B Strategy	Coordinates (q, p)
PSNE #1	Strategy 1	Strategy 1	(1, 1)
PSNE #2	Strategy 2	Strategy 2	(0, 0)

2. Mixed Strategy Nash Equilibrium:

✓ Mixed Strategy Nash Equilibrium:

Equilibrium Type	Player A Strategy	Player B Strategy	Coordinates (q, p)
Mixed Strategy Nash Equilibrium	Plays Strategy 1 with 0.625	Plays Strategy 1 with 0.375	(0.375, 0.625)

3. Display BRF on a unit square:



4. Short Report:

- ❑ There are two pure strategy equilibria at $(1,1)$ and $(0,0)$, where both players fully commit to one strategy.
- ❑ A mixed strategy equilibrium exists at $(0.375, 0.625)$, meaning neither player wants to unilaterally change their randomized behavior.
- ❑ **Strategic insight:** This game is symmetric, and players can benefit from mixing when avoiding predictable behavior.